STA500 Introduction to Probability and Statistics 2, autumn 2018.

Exercise set 1

Textbook, 9. edition: Exercises 3.42, 3.68, 4.55, 4.62, 4.63 and 4.90.
Textbook, 8. edition: Exercises 3.42, 3.70, 4.53, 4.64, 4.65 and 4.92.

Exercise 1:

A continuous random variable X has pdf (probability density function)

$$f(x) = \begin{cases} 4x(1-x^2) & \text{,for } 0 \leq x \leq 1 \\ 0 & \text{,otherwise.} \end{cases}$$

a) Calculate $P(X < 0.5)$, $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

b) Find the cumulative distribution function (cdf), $F(x)$, and use this to calculate $P(X < 0.3)$.

Exercise 2:

Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} k(1-x^2) & \text{for } -1 \leq x \leq 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

a) Determine k and make a sketch of $f(x)$.

b) Calculate $P(X \leq 0.5)$ and $P(X \leq 0.8 | X > 0.5)$.

c) Let $Y = 1 + x^2$. Calculate $\mathrm{E}(Y)$.

Exercise 3:

The gas price on different days at to gas stations in Sandnes has been registered. Let X be the gas price at one of the stations and let Y be the gas price on the other station on a random day. Registrations of X and Y on 10 randomly chosen days are shown in the table below.

dag i	1	2	3	4	5	6	7	8	9	10
x_i	13.89	13.39	12.20	14.35	14.10	13.39	13.96	14.15	13.69	12.57
y_i	13.99	13.39	12.65	14.25	13.99	13.09	13.66	14.25	13.36	12.57

It is given that $\sum_{i=1}^{10}(x_i-\bar{x})^2 = 4.436$, $\sum_{i=1}^{10}(y_i-\bar{y})^2 = 3.414$ and $\sum_{i=1}^{10}(x_i-\bar{x})(y_i-\bar{y}) = 3.670$.

a) Plot the gas prices at the two stations against each other on a scatter plot.

b) Calculate the empirical correlation between the gas prices and comment the result.

Exercise 4:

Let $X_1, \cdots, X_n$ be n independent random variables which all have the same expectation μ and variance σ^2. Let further $a_1, \cdots, a_n$ be constants, and define

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i, \qquad Y = \sum_{i=1}^{n} a_i X_i \qquad \text{and} \qquad Z = \frac{\sum_{i=1}^{n} a_i X_i}{\sum_{i=1}^{n} a_i}$$

a) Calculate $\mathrm{E}(\bar{X})$, $\mathrm{Var}(\bar{X})$, $\mathrm{E}(Y)$, $\mathrm{Var}(Y)$, $\mathrm{E}(Z)$ and $\mathrm{Var}(Z)$.

Some answers:

3.42: 0.632; **3.68/3.70**: c) 2/3; **4.55/4.53**: 0.8; **4.62/4.64**: 68;
4.63/4.65: 52; **4.90/4.92**: 0.258;
1: a) 0.4375 and 0.533, b) 0.1719; **2**: 0.75, 0.8438, 0.821 and 1.2; **3**: 0.94.