Exercise set 11

Note on Markov processes: Exercise 10,11,12,13

Exercise 1:

The lifetime of valves of various types are often modelled by a Weibull distribution. Recall that the density of the Weibull distribution can be written:

$$f(t) = \alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}}\ , \quad t \geq 0,$$

a) Find the hazard rate. Assume that $\alpha = 1$ and make a rough sketch of the hazard rate for $\beta = 0.5$, $\beta = 1.0$,$\beta = 1.5$. Comment the ageing properties for valves with each of these hazard rates (see also pages 204-205 in Walpole, Myers, Myers, Ye).

b) Still assume that $\alpha = 1$. Calculate $P(T > 2)$ and $P(T > 4|T > 2)$ for $\beta = 0.5$, $\beta = 1$ and $\beta = 1.5$. Compare with the results in a) and comment on ageing properties for different β values.

Exercise 2:

Suppose that the queue to be served at Idsø (a famous butcher in Stavanger) on a Saturday is a $M/M/4$ queue with arrival rate $\lambda = 1.5$ per minute and expected service time of $1/\gamma = 2$ minutes. When you arrive four persons are being served and 10 are waiting in line.

a) Explain why the queue is stable. Would the queue still be stable if one of the four persons serving customers had to leave work?

b) What is the expected waiting time until you start to get served? What is the expected waiting time until you are finished?

c) What is the probability that you will finish before the person in front of you in the queue? What is the probability that the person behind you will finish before you?

Exercise 3:

A machine can be in any of the three states; 0: normal, 1: degraded and 2: failed. Let $\{X(t) : t \geq 0\}$ be the state the machine is in at time t. Assume that this process is a continuous time Markov chain. Further assume that the expected time the machine stays in state 0 is 5 (months), the expected time in state 1 is 0.25 and the expected time in state 2 is 0.5. Moreover, when the machine changes state it is with the following probabilities: $p_{01} = 0.5$, $p_{02} = 0.5$, $p_{10} = 0.75$, $p_{12} = 0.25$ and $p_{20} = 1$.

a) What does it mean in practice that the state of the machine is a continuous time Markov chain? When we have observed that the machine has be functioning normally for 2 months, what is the probability that it will stop functioning normally (i.e. go to state 1 or 2) during the next month?

(Wait with point b) below until you have learned about steady state probabilities for continuous time Markov chains.)

b) Find the steady state probabilities for the process. In the long run, how much of the time is the machine functioning normally?

Some answers:

Note on Markov processes, 10 0.5,0,1.

11 0.5,17/40.

12 100/3.

13 0.66.

1 b) 0.243, 0.557, 0.135, 0.135, 0.059, 0.0057;

2 a) no; b) 5.5 and 7.5; c) 3/8 and 3/8;

3 a) 0.18; b) $\pi_0 = 80/87$, $\pi_1 = 2/87$,$\pi_2 = 5/87$;