STA500 Introduction to Probability and Statistics 2, autumn 2018.

Exercise set 2

Textbook, 9. edition: Exercises 5.58*, 5.61, 5.92 and 6.15.
Textbook, 8. edition: Exercises 5.60*, 5.65, 5.96 and 6.11.

$*$) Hint: The table of Poisson probabilities can be useful in this exercise. (You do not need to use the table, but you can save some time by using it in this and some of the other exercises.)

Exercise 1:

At a hospital unit 25% of the patients catch a certain kind of infection while they are at the hospital. Assume that all patients have the same probability of catching the infection and that the patients are infected or not, independent of each other. For the moment there are 12 patients at the unit. Let X denote the number of these 12 patients that will catch the infection.

a) Explain why X has a binomial distribution with parameters $n = 12$ and $p = 0.25$.

b) Calculate the probability that no patients are infected and the probability that more than 1 patient catch the infection.

c) Calculate the expectation and the standard deviation of the number of patients that will catch the infection.

Exercise 2:

The number of accidents at a certain road per year is assumed to be Poisson distributed with parameter $\lambda = 2$.

a) Calculate the probability that there will be exactly 2 accidents during one year. Calculate the probability that there will be at least 3 accidents during one year.

b) Calculate the probability that there will be exactly 2 accidents during half a year.

c) Calculate the probability that there will be at least 20 accidents during ten years.

d) Calculate the probability that there will be more than 2 accidents on this road during a year when we know that there has been at least one accident.

Exercise 3:

The weight of a certain type of bread has a normal distribution with expectation 750 grams and standard deviation 25 grams. You are buying two breads of this type. Let X_1 and X_2 denote the weight of these two breads. We assume that X_1 and X_2 are independent. Calculate

$$P(X_1 > 800), \quad P(X_1 \leq 800 | X_2 > 750) \quad \text{and} \quad P(X_1 + X_2 > 1600)$$

Exercise 4:

The carrying capacity for a certain kind of beams, measured as how many tons of load a beam can carry before it collapses, is known to have a normal distribution with expectation $\mu = 8.2$ and standard deviation $\sigma = 0.4$. Let X be the carrying capacity of a randomly chosen beam.

a) Show that the probability that a beam has a carrying capacity of less than 7.5 tons is 0.04.
If a beam has been tested and found to be capable of carrying a load of 7.5 tons without collapsing, what is the probability that this beam is having a carrying capacity of more than 8.2 tons? I.e. what is $P(X > 8.2 | X > 7.5)$?

We need 20 beams of the type considered in a particular construction. We set as a requirement that all these beams must have a carrying capacity of at least 7.5 tons. To assure that this requirement is fulfilled we expose new beams to 7.5 ton load until we have found 20 beams that can take this load without collapsing. We assume that whether each beam tested can take a load of 7.5 tons or not is independent of the remaining beams. Let Y be the number of beams we need to test until we have found 20 beams that is having a carrying capacity of at least 7.5 tons.

b) What is the distribution of Y? Explain.
Calculate $P(Y < 22)$.
Find $\mathrm{E}(Y)$.

Some answers:

5.58/5.60: a) 0.1512, b) 0.4015; **5.61/5.65**: a) 0.2657;
5.90/5.94: a) 0.1877, b) 0.0188 or 0.075; **5.92/5.96**: 0.1875;
6.15/6.11: a) 0.0571, b) 99.11%, c) 0.3974, d) 27.95, e) 0.0092;
1 b) 0.032 and 0.841, c) 3 and 1.5; **2** a) 0.271 and 0.323, b) 0.184, c) 0.5438, d) 0.373;
3 0.0228, 0.9772 and 0.0023 **4** a) 0.04 and 0.52, b) 0.796 and 20.8