STA500 Introduction to Probability and Statistics 2, autumn 2018.

Exercise set 3

Textbook, 9. edition:	Exercises 6.40, 6.43a), 6.47, 6.56, 6.57, 6.58 and 6.59.
Textbook, 8. edition:	Exercises 6.40, 6.43a), 6.47, 6.54, 6.55, 6.56 and 6.57.
Note on extreme values:	Exercise 1, 2

Exercise 1:

Let X be exponentially distributed with density function:

$$f(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta} & \text{,for } x \geq 0 \\ 0 & \text{,otherwise} \end{cases}$$

a) Show that $\mathrm{E}(X) = \beta$ and $\mathrm{Var}(X) = \beta^2$.

On the wrapping of a light bulb it is stated that the light bulb have an expected lifetime of 1000 hours. Assume that the lifetime of the bulb is exponentially distributed.

b) Calculate the probability that the bulb will have a lifetime of more than 1000 hours.

Exercise 2:

A secretary receives telephone calls frequently during the working day. Assume that the arrival of telephone calls to the secretary can be modeled as a Poisson process with parameter $\lambda = 6$ per hour. (Also assume that the secretary is having a telephone which record all calls, also new calls arriving when he is talking in the telephone.)

a) What is the probability that the secretary receives more than 6 telephone calls during one hour?

b) When the secretary arrives at the office in the morning, what is the probability that less than 10 minutes will elapse until the first time the telephone rings?

c) What is the probability that less than 20 minutes will elapse until the second time the telephone rings?

d) What is the probability that the secretary will receive more than 50 telephone calls during one working day of 7.5 hours?

e) The secretary is out of the office for 10 minutes. What is the probability that there has been no telephone calls while he was out?

f) The secretary returns after having been out of the office for 10 minutes and he sees that there has been no telephone calls while he was out. What is then the probability that at least another 10 minutes will elapse until the telephone rings?

Exercise 3:

The distribution of the time until failure (in years), X, for valves of a certain type is given by the probability density

$$f(x) = 0.02xe^{-0.01x^2} \qquad x \geq 0$$

Three valves of this type are being used in a system, and this system only works if all three valves work. Thus we have that the time until failure for the entire system is $U = \min(X_1, X_2, X_3)$. We assume that the times until failure for the three valves are independent.

a) Find the distribution of U.

b) Find $P(X < 5)$ and $P(U < 5)$.

c) Find $E(X)$ and $\mathrm{E}(U)$.
(Hint: Do you recognize any specific type of distribution?)

Some answers:

6.40 0.199; **6.43** a) 6 og 18; **6.47** a) 1.2533, b) 0.1353; **6.56/6.54** 0.2119; **6.57/6.55** e^6 og $e^{12}(e^4 - 1)$;

6.58/6.56 a) 0.0137, b) 0.458; **6.59/6.57** a) 0.0067, b) 0.2 ;

E1 $2\lambda(e^{-\lambda v} - e^{-2\lambda v})$ og $\frac{3}{2\lambda}$: **E2** Weibull, 200, $\frac{200}{n^2}$

1 b) 0.368: **2** a) 0.3937, b) 0.6321, c) 0.5940, d) 0.2061, e) 0.3679, f) 0.3679

3 b) 0.221 og 0.528; c) 8.86 and 5.12: