University of Stavanger

EXAM IN: STA500 INTRODUCTION TO PROBABILITY AND STATISTICS 2

DURATION: 4 HOURS DATE: DECEMBER 12, 2016

PERMITTED AIDS: Approved simple calculator (HP30S, Casio FX82, TI-30, Citizen SR-270X, Texas BA II Plus or HP17bII+).
One yellow A4 size sheet with handwritten notes is allowed.
Both sides of the sheet can be used.

THE EXAM CONSISTS OF 6 PROBLEMS ON 4 PAGES, 9 PAGES OF ENCLOSURES.

COURSE RESPONSIBLE: Jörn Schulz PHONE:

Problem 1: A message of the form "yes" is transmitted from mouth to mouth. In each transition the message "yes" is distorted into "no" with probability 0.4 and into "yes and no" with probability 0.1. Moreover, "no" is distorted into "yes" with probability 0.3 and into "yes and no" with probability 0.2. Finally, "yes and no" is distorted into "yes" with probability 0.4 and into "no" with probability 0.4. This system is described by a homogeneous Markov chain $\{X_n, n = 1, 2, \ldots\}$ with state space $S = \{0 = \text{“yes”}, 1 = \text{“yes and no”}, 2 = \text{“no”}\}$.

a) Draw the transition graph of the Markov chain and write down the transition probability matrix.
What is the period? Is this Markov chain irreducible?
What is the probability that the message is not "yes" after one transition, given that it was "yes" before the transition?
What is $P(X_4 = 0|X_0 = 0, X_1 = 1, X_2 = 1, X_3 = 2)$?

In the following, assume a different transition probability matrix, namely

$$P = \begin{pmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{pmatrix}.$$

b) How high is the probability that the recipient will receive the message "yes" after two transitions?
What is the probability of $P(X_0 = 0 \cap X_2 = 1 \cap X_4 = 1)$?
What are the steady state probabilities?

<u>Problem 2:</u> In every day life we receive in our mailbox two kinds of emails: acceptable emails and spam-mails. In the following, we call acceptable emails just emails. Let us assume that we receive an email with probability $p \in (0, 1)$ and that received emails and spam-mails are independent from each other. Let S_n be the number of spam-mails of n received messages.

a) What is the distribution of S_n? Explain your answer!
Suppose $p = 0.7$ and $n = 10$. What is the probability that we receive more than one spam mail?
What is the expected number of spam-mails $E(S_n)$ and the variance $Var(S_n)$?

Assume again, $p \in (0, 1)$. Now, let T_k be the random number of spam-mails until we receive the k'th acceptable email.

b) What is the distribution of T_2?
Suppose $p = 0.7$. What is the expected number of spam-mails until we receive the 2'nd acceptable email? What is the probability of $P(T_k \geq 2)$?

<u>Problem 3:</u> The yields on shares (avkastning av askjer) can, for example, be modeled using the the Laplace distribution. The distribution has a parameter λ and the following density function

$$f(x; \lambda) = \frac{1}{2\lambda} \exp(-\frac{|x|}{\lambda}), \quad x \in \mathbb{R}, \lambda > 0,$$

where $|x|$ is the absolute value of x. The parameter λ can be understood as the expected value of the absolute yields on shares $|X|$, i.e., $E(|X|) = \lambda$.

a) Supposed we have n independent observations $x_i, i = 1, \ldots, n$ of yields of one share. Show that the maximum likelihood estimator (MLE) of λ is given by

$$\hat{\lambda} = \frac{1}{n} \sum_{i=1}^{n} |x_i|.$$

Show further that $\hat{\lambda}$ is an unbiased estimator.
Assume we observe day's yields of $1.3, -0.6, 0.2, 0.4, -0.8$ during one week. Calculate the MLE $\hat{\lambda}$ for this sample and the Wald confidence interval.

<u>Problem 4:</u> Suppose we have two independent random variables Y_1 and Y_2 with $Y_1 \sim Poisson(\alpha\beta)$ and $Y_2 \sim Poisson((1 - \alpha)\beta)$. Suppose further that our prior information for α and β can be expressed as $\alpha \sim Beta(p, q)$ and $\beta \sim Gamma(p+q, 1)$ with α and β independent, for specified hyper-parameters p and q.

a) What is the joint posterior distribution of α and β and what are the marginal posterior distributions?

Problem 5: A company wants to quantify their carbon steel production. Therefore they take independent measurements of the yield point (på norsk flytespenning) of 10 carbon steel samples resulting in the following values ($1 Megapascal = 1 N/mm^2$):

$$x_1 = 332, \quad x_2 = 354, \quad x_3 = 338, \quad x_4 = 340, \quad x_5 = 345$$
$$x_6 = 360, \quad x_7 = 366, \quad x_8 = 352, \quad x_9 = 346, \quad x_{10} = 342.$$

It is assumed that the yield point is normally distributed and that a significance value of $\alpha = 0.05$ is given.

a) Estimate the expected yield point μ.
What is the confidence interval (CI) in case the variance is known to be $\sigma^2 = 105$?
What is the CI in case the variance is unknown?

b) Are the following two statements wrong or correct? Why?

- The true value μ is with probability 0.95 contained in the calculated CI for known variance.
- $\frac{1}{n}\sum_{i=1}^{n} x_i$ is with 95% probability in the interval calculated for known variance.

Assume it is known that the variance is $\sigma^2 \leq 105$. How large do we have to choose the sample size in order to obtain a confidence interval with maximal length $8 N/mm^2$ for μ?

Problem 6: A ticket office is selling tickets for an open-air-festival in the future. The customer arrival process can be described by Poisson-process with an average arrival rate of one customer per minute. Moreover, the ticket office needs 45 seconds for the ticket sale in average.

a) When you arrive one person is being served and 9 persons are waiting in line. Two question arise to you:

- What is the expected waiting time in minutes until you get served?
- You are in hurry and have only 6 minute left in order to buy a ticket. What is the probability that you will buy a ticket within 6min?

Draw the transition graph with the corresponding specific transition rates.

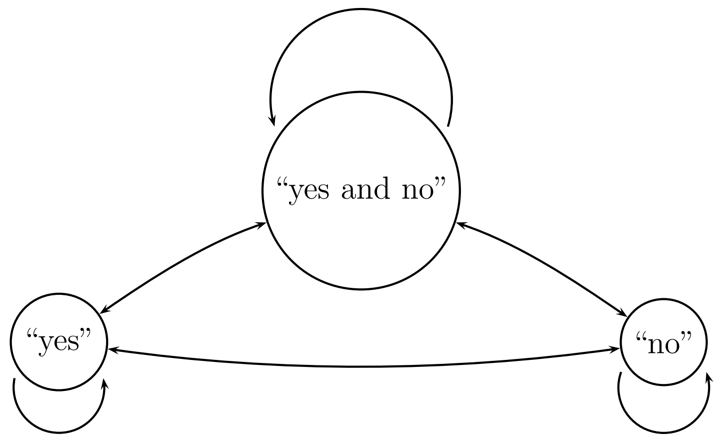
We have shown in the lecture that the steady state probabilities for the number of people in this queuing system can be described by (you **don't** have to show this again!)

$$\pi_k = (\frac{\lambda}{\gamma})^k \left(1 - \frac{\lambda}{\gamma}\right), \quad k = 0, 1, 2, 3, 4 \ldots.$$

b) Explain briefly why we have steady state probabilities for this Markov chain.
What is the expected number of people in this queuing system?
What is the probability that more than 3 people are waiting in the line?

Solutions

1.a
The transition graph is



and the transition matrix becomes

$$P = \begin{pmatrix} 0.5 & 0.1 & 0.4 \\ 0.4 & 0.2 & 0.4 \\ 0.3 & 0.2 & 0.5 \end{pmatrix}.$$

From the transition graph we see that all states communicate with each other, i.e., we have one class $\{0, 1, 2\}$ and thus this Markov chain is irreducible. Each state has a positive probability to return to itself in one transition, i.e., $p_{00} > 0, p_{11} > 0, p_{22} > 0$. Therefore, the period is one, i.e., the Markov chain is aperiodic. In addition, we have a finite state space. Thus, the Markov chain has steady state probabilities.
The probability that the message is not "yes" anymore after one transition is $p_{01} + p_{02} = 0.1 + 0.4. = 0.5$ and
$P(X_4 = 0|X_0 = 0, X_1 = 1, X_2 = 1, X_3 = 2) = P(X_4 = 0|X_3 = 2) = 0.3$.

1.b
Let us first calculate all two-steps transitions:

$$P^2 = P \cdot P = \begin{pmatrix} 0.35 & 0.36 & 0.29 \\ 0.26 & 0.40 & 0.34 \\ 0.26 & 0.36 & 0.38 \end{pmatrix}.$$

The probability that the recipient will receive the message "yes" after two transitions is $p_{00}^2 = 0.35$. We have

$$\begin{aligned} P(X_0 = 0 \cap X_2 = 1 \cap X_4 = 1) &= P(X_0 = 0)P(X_2 = 1 \mid X_0 = 0)P(X_4 = 1 \mid X_2 = 1) \\ &= 1 p_{01}^2 p_{11}^2 = 0.36 \cdot 0.4 = 0.144. \end{aligned}$$

Finally, the steady state probability are

$$\begin{aligned}
& \Pi = P^T \Pi \\
\Leftrightarrow \quad & \begin{pmatrix} \pi_0 \\ \pi_1 \\ \pi_2 \end{pmatrix} = \begin{pmatrix} 0.5 & 0.2 & 0.2 \\ 0.3 & 0.5 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{pmatrix} \begin{pmatrix} \pi_0 \\ \pi_1 \\ \pi_2 \end{pmatrix} \\
\Leftrightarrow \quad & \pi_0 = 0.5\pi_0 + 0.2\pi_1 + 0.2\pi_2 \\
& \pi_1 = 0.3\pi_0 + 0.5\pi_1 + 0.3\pi_2 \\
& \text{plus the condition } \pi_2 = 1 - \pi_0 - \pi_1 \\
\Leftrightarrow \quad & \pi_0 = 0.5\pi_0 + 0.2\pi_1 + 0.2(1 - \pi_0 - \pi_1) \\
& \pi_1 = 0.3\pi_0 + 0.5\pi_1 + 0.3(1 - \pi_0 - \pi_1) \\
\Leftrightarrow \quad & 0.7\pi_0 = 0.2 \Leftrightarrow \pi_0 = \frac{0.2}{0.7} = \frac{2}{7} \\
& 0.8\pi_1 = 0.3 \Leftrightarrow \pi_1 = \frac{0.3}{0.8} = \frac{3}{8} \\
& \pi_2 = 1 - \frac{2}{7} - \frac{3}{8} = \frac{56}{56} - \frac{16}{56} - \frac{21}{56} = \frac{19}{56}.
\end{aligned}$$

The steady state probabilities are

$$\begin{pmatrix} \pi_0 \\ \pi_1 \\ \pi_2 \end{pmatrix} = \begin{pmatrix} \frac{2}{7} \\ \frac{3}{8} \\ \frac{19}{56} \end{pmatrix} \approx \begin{pmatrix} 0.2857 \\ 0.375 \\ 0.3393 \end{pmatrix}.$$

2.a

We receive n independent messages, i.e., we have n independent trails. Further, we classify the messages into accepted-emails ("success") or spam-mails ("no success"). The probability is $q = 1 - p$ for a spam-mail (i.e. "no success") is the same in each trial. Moreover, we receive n messages. In this situation, S_n="the number of spam-mails of n received messages" is binomial distributed with $S_n \sim \mathrm{B}(n, q)$, i.e.,

$$P(S_n = k) = \binom{n}{k} q^k p^{n-k}, \quad k = 0, \ldots, n.$$

Moreover,

$$\begin{aligned}
P(S_n > 1) = P(S_n \geq 2) &= 1 - P(S_n = 0) - P(S_n = 1) = 1 - q^0 p^{10} - \binom{10}{1} q p^9 \\
&= 1 - 0.7^{10} - 10 * 0.3 * 0.7^9 \approx 0.8507,
\end{aligned}$$

i.e., we have a probability of 85% that there are at least two spam mails if we receive 10 messages. Moreover, $E(S_n) = nq = 10 * 0.3 = 3$ and $Var(S_n) = nqp = 10 * 0.3 * 0.7 = 2.1$.

2.b

Now, let T_2 be the random number of spam-mails until we receive the 2nd acceptable-email. Further, let X_2 be the random number of messages (trials) until we receive the 2nd acceptable-email, i.e., $X_2 = T_2 + 2$. , i.e.,

$$\begin{aligned} P(T_2 = j) &= P(X_n = j+2) \\ &= P(\text{the first } j+1 \text{ messages contains exact } j \text{ spam-mails} \\ &\qquad \text{and one acceptable-email, afterwards a second acceptable-email follows}) \\ &= P(S_n = j) \cdot P(\text{acceptable-email}) \\ &= \binom{j+1}{j} q^j p \cdot p = (j+1)p^2 q^j, \end{aligned}$$

i.e. T_2 is negative binomial distributed shifted by -2 compared to the form in the collection of formulas. Therefore, the expected number of spam-mails until we receive the second acceptable-email is

$$\begin{aligned} E(T_2) &= E(X_2 - 2) = E(X_2) - 2 = \frac{2}{p} - 2 = 2\frac{1-p}{p} \\ &= 2\frac{1-0.7}{0.7} = 2\frac{3}{7} = \frac{6}{7} \approx 0.857. \end{aligned}$$

Moreover, we have

$$\begin{aligned} P(T_2 \geq 2) &= 1 - P(T_2 = 0) - P(T_2 = 1) = 1 - (0+1)p^2q^0 - (1+1)p^2q^1 \\ &= 1 - 0.7^2 - 2(0.7)^2 0.3 = 0.216. \end{aligned}$$

3.a

The likelihood function is given by

$$L(\lambda) = \prod_{i=1}^{n} f(x_i; \lambda) = \prod_{i=1}^{n} \frac{1}{2\lambda} \exp\left(-\frac{|x_i|}{\lambda}\right).$$

The the log-likelihood is

$$\begin{aligned} l(\lambda) &= \ln\left(\prod_{i=1}^{n} \frac{1}{2\lambda} \exp\left(-\frac{|x_i|}{\lambda}\right)\right) = \sum_{i=1}^{n} \ln\left(\frac{1}{2\lambda} \exp\left(-\frac{|x_i|}{\lambda}\right)\right) \\ &= \sum_{i=1}^{n} \left(-\ln(2\lambda) - \frac{|x_i|}{\lambda}\right) \end{aligned}$$

and further

$$\begin{aligned} \frac{d}{d\lambda} l(\lambda) &= \sum_{i=1}^{n} \left(-\frac{2}{2\lambda} + \frac{|x_i|}{\lambda^2}\right) \\ &= -\frac{n}{\lambda} + \frac{1}{\lambda^2} \sum_{i=1}^{n} |x_i| \end{aligned}$$

and

$$\begin{aligned} & \frac{d}{d\lambda}l(\lambda) = -\frac{n}{\lambda} + \frac{1}{\lambda^2}\sum_{i=1}^{n}|x_i| = 0 \\ \Leftrightarrow \quad & -n\lambda + \sum_{i=1}^{n}|x_i| = 0 \\ \Leftrightarrow \quad & \lambda = \frac{1}{n}\sum_{i=1}^{n}|x_i|. \end{aligned}$$

The MLE candidate is $\hat{\lambda} = \frac{1}{n}\sum_{i=1}^{n}|x_i|$. Have to check the second derivative:

$$J(\lambda) = \frac{d}{d\lambda^2}l(\lambda) \quad = \quad \frac{n}{\lambda^2} - \frac{2}{\lambda^3}\sum_{i=1}^{n}|x_i| = \frac{n}{\lambda^2} - \frac{2}{\lambda^3}n\hat{\lambda}$$

and

$$J(\hat{\lambda}) \quad = \quad \frac{n}{\hat{\lambda}^2} - \frac{2}{\hat{\lambda}^3}n\hat{\lambda} = \frac{1}{\hat{\lambda}^2}\left(n - \frac{2}{\hat{\lambda}}n\hat{\lambda}\right) = -\frac{n}{\hat{\lambda}^2} < 0.$$

Thus, $\hat{\lambda}$ is a maximixer and therewith $\hat{\lambda}$ is a MLE. Moreover, we have

$$E(\hat{\lambda}) = E\left(\frac{1}{n}\sum_{i=1}^{n}|x_i|\right) = \frac{1}{n}\sum_{i=1}^{n}E(|x_i|) = \frac{1}{n}\sum_{i=1}^{n}\lambda = \frac{1}{n}n\lambda = \lambda$$

which means $\hat{\lambda}$ is an unbiased estimator. Given the sample, we get

$$\hat{\lambda} = \frac{1}{n}\sum_{i=1}^{n}|x_i| = \frac{1}{5}\left(|1.3| + |-0.6| + |0.2| + |0.4| + |-0.8|\right) = 0.66.$$

For the Wald confidence interval, we have $Var(\hat{\lambda}) = -\frac{1}{J(\hat{\lambda})} = \frac{1}{n}\hat{\lambda}^2 = 0.08712$ which leads to the Wald-interval

$$\hat{\lambda} \pm z_{0.025}\sqrt{Var(\hat{\lambda})} = 0.66 \pm 1.96 \cdot 0.2952,$$

i.e. $(0.08, 1.24)$. We can conclude that we need a larger sample size in order to be more confident about the estimate $\hat{\lambda}$.

4.a

Because Y_1 and Y_2 are independent, $Y_1 \sim Poisson(\alpha\beta)$ and $Y_2 \sim Poisson((1-\alpha)\beta)$ we have the following likelihood,

$$f(y_1, y_2|\alpha, \beta) = \frac{(\alpha\beta)^{y_1}}{y_1!}\exp(-\alpha\beta) * \frac{((1-\alpha)\beta)^{y_2}}{y_2!}\exp(-(1-\alpha)\beta).$$

Further, because of α and β are independent, $\alpha \sim Beta(p, q)$, $\beta \sim Gamma(p+q, 1)$,

$$\pi(\alpha, \beta) = \pi(\alpha)\pi(\beta) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)}\alpha^{p-1}(1-\alpha)^{q-1} * \frac{1}{\Gamma(p+q)}\beta^{p+q-1}\exp(-\beta).$$

Then the joint posterior distribution is given by

$$\begin{aligned} f(\alpha, \beta | y_1, y_2) &\propto e^{\beta}\beta^{y_1+y_2}\alpha^{y_1}(1-\alpha)^{y_2}\alpha^{p-1}(1-\alpha)^{q-1}e^{\beta}\beta^{p+q-1} \\ &= \beta^{y_1+y_2+p+q-1}e^{-2\beta}\alpha^{y_1+p-1}(1-\alpha)^{y_2+q-1}. \end{aligned}$$

This joint probability contains all information from the data and the prior. In this particular case, the posterior factories into functions of α and β. Therefore, the marginal distributions are given by

$$\begin{aligned} f(\alpha | y_1, y_2) &= \int_0^\infty f(\alpha, \beta | y_1, y_2) d\beta \propto \alpha^{y_1+p-1}(1-\alpha)^{y_2+q-1}, \\ f(\beta | y_1, y_2) &= \int_0^1 f(\alpha, \beta | y_1, y_2) d\alpha \propto \beta^{y_1+y_2+p+q-1}e^{-2\beta}. \end{aligned}$$

That is, $\alpha | y_1, y_2 \sim Beta(y_1 + p, y_2 + q)$ and $\beta | y_1 + y_2 \sim Gamma(y_1 + y_2 + p + q, 2)$.

5.a

The standard estimator for μ is $\hat{\mu} = \bar{X} = \sum_{i=1}^{10} x_i = 347.5$. We have known variance with $\sigma^2 = 105$ and $\alpha = 0.05$. We know from the lecture

$$\frac{\sqrt{n}(\bar{X} - \mu)}{\sigma} \sim N(0, 1),$$

and therefore

$$\begin{aligned} & P\left(-z_{\frac{\alpha}{2}} \leq \frac{\sqrt{n}(\bar{X} - \mu)}{\sigma} \leq z_{\frac{\alpha}{2}}\right) = 1 - \alpha \\ \Leftrightarrow \quad & P\left(\bar{X} - \frac{\sigma}{\sqrt{n}} z_{\frac{\alpha}{2}} \leq \mu \leq \bar{X} + \frac{\sigma}{\sqrt{n}} z_{\frac{\alpha}{2}}\right) = 1 - \alpha, \end{aligned}$$

i.e. the confidence interval is

$$\begin{aligned} \left[\bar{X} - \frac{\sigma}{\sqrt{n}} z_{\frac{\alpha}{2}}, \bar{X} + \frac{\sigma}{\sqrt{n}} z_{\frac{\alpha}{2}}\right] &= \left[347.5 - \sqrt{\frac{105}{10}}1.96, 347.5 + \sqrt{\frac{105}{10}}1.96\right] \\ &= [341.15, 353.85]. \end{aligned}$$

In case the variance is unknown, we have to replace σ^2 by the unbiased estimator $s^2 = \frac{1}{10-1}\sum_{i=1}^{1} 0(X_i - \bar{X})^2 = 109.61$. In this case we know from the lecture,

$$\frac{\sqrt{n}(\bar{X} - \mu)}{s} \sim t_{n-1},$$

and therefore

$$\begin{aligned} & P\left(-t_{\frac{\alpha}{2},n-1} \leq \frac{\sqrt{n}(\bar{X}-\mu)}{s} \leq t_{\frac{\alpha}{2},n-1}\right) = 1-\alpha \\ \Leftrightarrow \quad & P\left(\bar{X} - \frac{s}{\sqrt{n}} t_{\frac{\alpha}{2},n-1} \leq \mu \leq \bar{X} + \frac{s}{\sqrt{n}} t_{\frac{\alpha}{2},n-1}\right) = 1-\alpha, \end{aligned}$$

i.e. the confidence interval is

$$\left[347.5 - \sqrt{\frac{109.61}{10}}2.26, 347.5 + \sqrt{\frac{109.61}{10}}2.26\right] = [340.00, 355.00] .$$

5.b

The first comment is wrong. The unknown parameter μ is either in the interval or not. The second comment is also wrong because

$$\frac{1}{n}\sum_{i=1}^{n} x_i$$

lies always in the confidence interval, namely exactly in the center.

Let l_{CI} be the length of the confidence interval (CI) in case the variance is known. We know from above $l_{CI} = 2\frac{\sigma}{\sqrt{n}} z_{1-\frac{1}{2}}$, and from the text that $\sigma^2 \leq 105$ and the maximal length of the CI for μ has to be $8N/mm^2$. Thus,

$$\begin{aligned} l_{CI} \quad \leq \quad & 2\frac{\sigma}{\sqrt{n}} z_{1-\frac{1}{2}} \leq 8 \\ \Leftrightarrow \quad & \sqrt{n} \geq 2\frac{\sqrt{105}}{8} 1.96 = 5.02 \\ \Leftrightarrow \quad & n \geq 25.2, \end{aligned}$$

i.e., we need a sample size of $n = 26$ in order to obtain a confidence interval smaller than $8N/mm^2$.

6.a

In the following calculation we choose hour as the time unit, i.e., we have the arrival rate is $\lambda = 60$ and the departure rate is $\gamma = 80$ given one server. The expected time W_{10} you have to wait until you get served can be derived from the expected serving time for each person in the front of you plus the remaining serving time for person who gets served when you arrive at the ticket office. Let $Z_i =$"the time between each time the queue moves forward". Because of the memoryless property of the exponential distribution, we know all Z_i's are exponential distributed with $E(Z_i) = \frac{1}{\gamma} = \frac{1}{80}$. Therefore we have

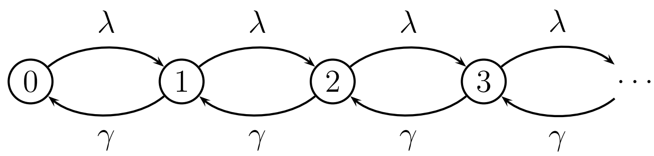
$$E(W_{10}) = E\left(\sum_{i=1}^{10} Z_i\right) = \sum_{i=1}^{10} E(Z_i) = 10\frac{1}{80} = \frac{1}{8},$$

i.e., the expected waiting time until you get served is $\frac{1}{8}$hour $= 7.5$min.
In order to buy a ticket within 6min, you and all 10 persons in the front of you have to get served (9 persons are waiting in line plus one person who is being served). This means, 11 people have to leave the system within 6min $= \frac{1}{10}$hour. Let the random variable N be the number of events in the interval $[0, 6]$min $= [0, \frac{1}{10}]$hour, i.e., the number of people who leave the system. We know, $N \sim Poisson(\gamma(b-a)) = Poisson(80\frac{1}{10}) = Poisson(8)$. Therefore,

$$P(N \geq 11) = 1 - P(N < 11) = 1 - P(N \leq 10) = 1 - 0.8159 = 0.1841,$$

(using the table for the Poisson distribution with $x = 10$ and $\mu = 8$) i.e., you will get only with 18% probability a ticket within 6min.

The transition graph with attached specific transition rates is given by:



6.b
Steady state probabilities exist because we have a stable queue ($\lambda < \gamma$) which is irreducible (all states communicate with each other), positive recurrent and aperiodic. Let K be the number of people in the queuing system. Then we have,

$$\begin{aligned} E(K) &= \sum_{k=0}^{\infty} kP(K=k) = \sum_{k=0}^{\infty} k\pi_k = \sum_{k=0}^{\infty} k(\frac{\lambda}{\gamma})^k \left(1 - \frac{\lambda}{\gamma}\right) \\ &= \left(1 - \frac{\lambda}{\gamma}\right) \sum_{k=0}^{\infty} k(\frac{\lambda}{\gamma})^k = \left(1 - \frac{\lambda}{\gamma}\right) \frac{\frac{\lambda}{\gamma}}{(1 - \frac{\lambda}{\gamma})} \\ &= \frac{\frac{\lambda}{\gamma}}{1 - \frac{\lambda}{\gamma}} = \frac{\lambda}{\gamma - \lambda} = \frac{60}{80 - 60} = 3, \end{aligned}$$

i.e., the expected number of people in this queuing system is 3.
The probability that more than 3 people are waiting in the line means that there are more than 4 people in the queuing system,

$$\begin{aligned} P(K > 4) &= 1 - P(K \leq 4) = 1 - \sum_{k=0}^{4} \pi_k \\ &= 1 - \left(1 - \frac{\lambda}{\gamma}\right) \sum_{k=0}^{4} (\frac{\lambda}{\gamma})^k = 1 - \left(1 - \frac{\lambda}{\gamma}\right) \frac{1 - (\frac{\lambda}{\gamma})^{4+1}}{1 - \frac{\lambda}{\gamma}} \\ &= 1 - \left(1 - (\frac{\lambda}{\gamma})^5\right) = \left(\frac{60}{80}\right)^5 = 0.237. \end{aligned}$$

There is a probability of 23.7% that more than 3 people waiting in the line. That means you were quiet unlucky when you arrived to the queue.