



UNIVERSITETET I STAVANGER

Det Teknisk-Naturvitenskapelige Fakultet

EKSAMEN I: MAF300 Numerisk Modelling, grunnkurs

DATO: 7. december 2017

DURATION: 4 HOURS

AUTHORIZED MATERIAL: All theoretical material from Canvas. Rao's *Applied Numerical methods for engineers and scientists*. Any calculator.

THERE ARE FOUR PROBLEMS ON 2 PAGES

Problem 1

The following function is defined for $x > 0$:

$$f(x) = \ln(x) + x^2.$$

Equation $f(x) = 0$ has one root, r , between 0 and 1.

Find r and write it, correctly rounded to six decimals.

It is expected that you state the method that you use, and that you give enough details that your calculations can be checked.

Problem 2

Number theory states that an estimate of the number of prime numbers between two positive integers a and b , is given by

$$\int_a^b \frac{dx}{\ln(x)}.$$

The purpose of this problem is to get such an estimate for $a = 100$ and $b = 200$, and using Romberg integration.

More precisely you are asked to calculate successive approximations of $\int_{100}^{200} dx / \ln(x)$, as follows:

1. Use the trapezoidal rule with 1, then 2 trapezes.
2. Perform a Richardson extrapolation.
3. Use the trapezoidal rule with four trapezes.
4. Perform two Richardson extrapolations.
5. Give the value of the integral with only correct decimals.

Problem 3

Function $y(x)$ is defined for $x \geq 0$. It satisfies the following initial value problem:

$$y'(x) = x + y^2, \quad y(0) = 0.$$

Find $y(0.5)$, and express your result correctly rounded to four decimals, using

1. Euler's method with step $h = 0.5$;
2. Fourth-order Runge-Kutta method with $h = 0.5$.

Problem 4

Four values of a function have been calculated, and the result is given in the following table

x	0.0000	0.5000	1.000	1.500
y	0.0000	0.1266	0.5572	1.786

Give the expression of the Newton interpolating polynomial that passes through the four points.

Problem 1

```
> restart;
```

```
> f:=x->ln(x) + x^2:
```

```
I use Newton-Raphson
```

```
> fp:=x->1/x+2*x:
```

```
> x0:=1.0; x1:=x0-f(x0)/fp(x0);
```

```
x0 := 1.0
```

```
x1 := 0.6666666667
```

(1)

```
> x2:=x1-f(x1)/fp(x1);
```

```
x2 := 0.6529092538
```

(2)

```
> x3:=x2-f(x2)/fp(x2);
```

```
x3 := 0.6529186404
```

(3)

```
> x4:=x3-f(x3)/fp(x3);
```

```
x4 := 0.6529186404
```

(4)

```
> f(x4);
```

```
0.
```

(5)

```
> restart;
```

Problem 2

Integrate $1/\ln(x)$ from 100 to 200. It is equal a bit more than 20.

Calculate $T(0,1)$ = area of trapeze with height 100; $T(0,2)$ = sum of 2 trapeze areas, ...

```
> f:=x->evalf(1./ln(x));
```

$$f := x \rightarrow \text{evalf}\left(\frac{1}{\ln(x)}\right) \quad (1)$$

```
> a:=100.: b:=200:
```

```
> T01:=0.5*(b-a)*(f(100)+f(200));
```

$$T01 := 20.29432034 \quad (2)$$

```
> T02:=0.5*((b-a)/2)*(f(100)+2*f(150)+f(200));
```

$$T02 := 20.12593473 \quad (3)$$

```
> T12:=4*T02/3-T01/3;
```

$$T12 := 20.06980619 \quad (4)$$

```
> T04:=0.5*((b-a)/4)*(f(100)+f(200)+2*add(f(100+k*100/4),k=1..3));
```

$$T04 := 20.08123012 \quad (5)$$

```
> T14:=4*T04/3-T02/3; abs(T14-T04);
```

$$T14 := 20.06632858 \\ 0.01490154 \quad (6)$$

```
> T24:=16*T14/15-T12/15; abs(T24-T14);
```

$$T24 := 20.06609674 \\ 0.00023184 \quad (7)$$

Stop here to answer the question:20.066

I continue just to see how things work

```
> T08:=0.5*((b-a)/8)*(f(100)+f(200)+2*add(f(100+k*100/8),k=1..7));
```

$$T08 := 20.06984492 \quad (8)$$

```
> T18:=4*T08/3-T04/3; abs(T18-T08);
```

$$T18 := 20.06604986 \\ 0.00379506 \quad (9)$$

```
> T28:=16*T18/15-T14/15; abs(T28-T18);
```

$$T28 := 20.06603128 \\ 0.00001858 \quad (10)$$

```
> T38:=(4^3)*T28/(4^3-1)-T24/(4^3-1); abs(T38-T28);
```

$$T38 := 20.06603024 \\ 0.00000104 \quad (11)$$

```
> T016:=0.5*((b-a)/16)*(f(100)+f(200)+2*add(f(100+k*100/16),k=1..15));
```

$$T016 := 20.06698438 \quad (12)$$

```
> T116:=4*T016/3-T08/3; abs(T116-T016);  
    T116 := 20.06603086  
    0.00095352 (13)
```

```
> T216:=16*T116/15-T18/15; abs(T216-T116);  
    T216 := 20.06602959  
    0.00000127 (14)
```

```
> T316:=(4^3)*T216/(4^3-1)-T28/(4^3-1); abs(T316-T216);  
    T316 := 20.06602957  
    2. 10-8 (15)
```

```
> T416:=(4^4)*T316/(4^4-1)-T38/(4^4-1); abs(T416-T316);  
    T416 := 20.06602957  
    0. (16)
```

```
> int(1/ln(x), x=100..200, numeric);  
    20.06602958 (17)
```

```
> restart;
```

Problem 3

```
> f := (x, y) -> x + y^2; x0 := 0; y0 := 0; h := 0.5;
```

$$f := (x, y) \rightarrow x + y^2$$

$$x0 := 0$$

$$y0 := 0$$

$$h := 0.5$$

(1)

Euler: $y_1 = y(h) = h * f(x_0, y_0)$

```
> y1 := y0 + h * f(x0, y0);
```

$$y1 := 0.$$

(2)

Fourth order Runge-Kutta

```
> k1 := f(x0, y0);
```

$$k1 := 0$$

(3)

```
> k2 := f(x0 + h/2, y0 + h*k1/2);
```

$$k2 := 0.2500000000$$

(4)

```
> k3 := f(x0 + h/2, y0 + h*k2/2);
```

$$k3 := 0.2539062500$$

(5)

```
> k4 := f(x0 + h, y0 + h*k3);
```

$$k4 := 0.5161170960$$

(6)

```
> y1 := y0 + h * (k1 + 2*k2 + 2*k3 + k4) / 6;
```

$$y1 := 0.1269941330$$

(7)

Try the Maple default solver dsolve

```
> equ := diff(y(x), x) = f(x, y(x));
```

```
> ini := y(0) = 0;
```

```
> sol := dsolve({equ, ini}, y(x), numeric, output=listprocedure);
```

```
> Y := eval(y(x), sol);
```

$$Y := \text{proc}(x) \dots \text{end proc}$$

(8)

```
> Y(0.5);
```

$$0.126587329597345$$

(9)

```
> Y(1.0);
```

$$0.557161776583682$$

(10)

```
> Y(1.5);
```

$$1.78569402225906$$

(11)

```
> restart;
```

Problem 4

```
> x0:=0.00:   y0:=0.0000:
   x1:=0.50:   y1:=0.1266:
   x2:=1.00:   y2:=0.5572:
   x3:=1.50:   y3:=1.786:
```

The Newton interpolating polynomial

```
> g1_x0 := y0:
   g1_x1 := y1:
   g1_x2 := y2:
   g1_x3 := y3:
   g2_x1x0 := (g1_x1 - g1_x0) / (x1-x0);
   g2_x2x1 := (g1_x2 - g1_x1) / (x2-x1);
   g2_x3x2 := (g1_x3 - g1_x2) / (x3-x2);
   g3_x2x1x0 := (g2_x2x1 - g2_x1x0) / (x2-x0);
   g3_x3x2x1 := (g2_x3x2 - g2_x2x1) / (x3-x1);
   g4_x3x2x1x0 := (g3_x3x2x1 - g3_x2x1x0) / (x3-x0);
                                     g2_x1x0 := 0.2532000000
                                     g2_x2x1 := 0.8612000000
                                     g2_x3x2 := 2.4576000000
                                     g3_x2x1x0 := 0.6080000000
                                     g3_x3x2x1 := 1.5964000000
                                     g4_x3x2x1x0 := 0.6589333333333333 (1)
> n:=x->g1_x0+g2_x1x0*(x-x0)+g3_x2x1x0*(x-x0)*(x-x1)+g4_x3x2x1x0*(x-
   x0)*(x-x1)*(x-x2):
> n(0);
                                     0. (2)
> n(0.5);
                                     0.1266000000 (3)
> n(1);
                                     0.5572000000 (4)
> n(1,5);
                                     0.5572000000 (5)
> n(x);
0.2532000000 x + 0.6080000000 x (x - 0.50) + 0.65893333333 x (x - 0.50) (x - 1.00) (6)
```